PROPAGATION IN A RECTANGULAR WAVEGUIDE PERIODICALLY LOADED WITH RESONANT IRISES

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ABSTRACT

A wide-band network model is derived for interacting resonant irises in rectangular waveguides by means of field and network theoretical considerations. The model is applied to the analysis of the propagation characteristics in a waveguide periodically loaded with such irises.

Introduction

Recently the corrugated waveguide has found wide use in radar and communication systems. Nevertheless, a potentially important configuration such as the rectangular waveguide periodically loaded with resonant irises has not been satisfactorily investigated.

In this paper, we present an approach, which makes joint use of field and network theoretical methods. A periodically loaded waveguide is modelled by means of a cascade of identical multiport reactances connected by a finite number of uncoupled transmission lines. Each transmission line represents an "accessible mode" of the discontinuity, i.e., a mode that "sees" the adjacent discontinuities in the cascade. This includes all propagating modes plus the first few evanescent ones.

The reactance matrix of the multiport network representing the discontinuity is obtained by means of a method described in the literature [1].

This yields a wide-band multiport equivalent network with frequency-independent elements so that the field problem is translated entirely into a network problem and repetition of the field analysis at each frequency point is no longer necessary. Numerically, the method has variational properties, and manipulations with small matrices only are involved. The unit cell of the periodic structure can now be modelled by means of a given lumped network between two sets of uncoupled transmission lines. Upon application of Floquet's theorem, the propagation constants and the field patterns of the modes of the periodic structure can be derived.

Apart from the application to the periodic waveguide, the information provided on the resonant iris is useful in the design of filters and impedance matching networks.

Field Formulation

The geometry of a single resonant iris is illustrated in Fig. 1. For simplicity we will treat the case of an infinitely thin and symmetric iris. The case of finite thickness can be treated by methods similar to those of [1] and [2]. For TE_{10} excitation, the family of LSE_{x-mn} modes with m odd and n even is excited. The relevant potentials are as follows:

$$\psi_i(x,y) = \int E_y dx = \frac{\sqrt{\varepsilon_n}}{m} \cos \frac{m\pi}{a} x \cos \frac{n\pi y}{b} \qquad (1)$$

with $\varepsilon_0 = \frac{1}{2}$, $\varepsilon_n = 1$ for $n \neq 0$; $i \leftrightarrow (m,n)$;

$$\int H_x d_x = y_{0,mn}\psi_i \qquad (2)$$

with
$$y_{0mn} = \frac{\left(\frac{m\pi}{a}\right)^2 - k_0^2}{j\omega\mu_0\gamma_{mn}} ; \qquad (3)$$

$$\gamma_{mn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - K_0^2} \quad . \qquad (4)$$

The Rayleigh-Ritz variational expression for the reactance of the iris is [1]

$$x_{ij} = Q_{i*} \cdot B^{-1} \cdot Q_{j*}^T \qquad (5)$$

where $1 \leq i,j \leq K$, K being the number of accessible modes, which we number according to increasing cutoff. Q_{i*} is the N-dimensional row vector representing the i^{th} accessible mode in terms of an appropriate orthonormal basis on the aperture; B is the N x N truncation of the infinite matrix representing the waveguide Green's function in the same basis on the aperture. From the solutions of the capacitive and inductive cases, it appears that a convenient choice of basis is provided by the "Schwinger functions [3]." These lead to the finite expansion

$$\cos \frac{m\pi x}{a} = \sum_{m=1,3,\ldots}^{m} P_{mp} \cos p\theta \ , \ 0 \leq \theta \leq \pi \quad . \qquad (6)$$

Similarly, in the y-direction

$$\cos \frac{n\pi y}{b} = \sum_{q=0,2,\ldots}^{n} S_{nq} \cos q\eta \ , \ 0 \leq \eta \leq \pi \quad . \qquad (7)$$

In the above representation, the matrix B takes the form

$$-jB_{hg} = \sum_{\ell>k} 2Q_{\ell h}Q_{\ell g}Y_{0\ell} \qquad (8)$$

where

$$Q_{\ell h} = \frac{\sqrt{\varepsilon_n}}{m\sqrt{\varepsilon_q}} P_{mp}S_{nq} \ ; \ h \leftrightarrow (p,q) \qquad (9)$$

$$\ell \leftrightarrow (m,n)$$

$$Q_{\ell g} = \frac{\sqrt{\varepsilon_n}}{m\sqrt{\varepsilon_s}} P_{mr}S_{ns} \ ; \ g \leftrightarrow (r,s) \quad . \qquad (10)$$

Frequency Dependence

In order to obtain a wide-band equivalent network, we will now extract the frequency dependence of the reactance matrix.

The normalized propagation constant $\bar{\beta} = \left[\left(\frac{2d}{\lambda_0}\right)^2 - 1\right]^{1/2}$ provides an expedient frequency variable for this problem. The characteristic admittance, normalized to that of the fundamental mode, is

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$$\bar{Y}_{0,i}(j\bar{\beta}) = \frac{m^2 - \bar{\beta}^2 - 1}{j\bar{\beta}\sqrt{m^2 + \left(\frac{a}{b}n\right)^2 - \bar{\beta}^2 - 1}} \quad . \tag{11}$$

For $n > 0$ and $m > 1$, the above distributed admittance can be very closely approximated by means of the lumped admittance

$$y_{0,i}(j\bar{\beta}) = \frac{K_1^{(i)}}{j\bar{\beta}}\,\frac{m^2 - 1}{c_i} + \frac{k_2^{(i)} j\bar{\beta}}{c_i} \tag{12}$$

$$c_i = \sqrt{m^2 + \left(\frac{a}{b}n\right)^2 - 1} \tag{13}$$

the constants K_1, K_2 are close to unity and tend to unity as $m,n \to \infty$. In fact, they are determined as

$$\min_{K_1,K_2} \max |\bar{Y}_{0,i} - y_{0,i}| \tag{14}$$

for $\bar{\beta}$ in the band of interest.

The case $m = 1, n = 0$ is treated similarly. Introducing (12) in (5), the inversion of B can be carried out formally and we obtain the reactance matrix X in its Foster canonical form [1]

$$\sum_{v=1}^{N} r_{ij}^{(v)}\bar{\beta}\left[1 - (\bar{\beta}/\bar{\beta}_v)^2\right]^{-1} \tag{15}$$

N being the order of the variational solution.

The Corrugated Waveguide

The unit cell of the periodic structure and its network model are depicted in Fig. 2. The transfer matrix of the unit cell is given by

$$L = \begin{pmatrix} T & U \\ V & T \end{pmatrix} = \begin{pmatrix} Ch & ZSh \\ Z^{-1}Sh & Ch \end{pmatrix}\begin{pmatrix} I & 0 \\ -jX^{-1} & 1 \end{pmatrix}\begin{pmatrix} Ch & ZSh \\ Z^{-1}Sh & Ch \end{pmatrix} \tag{16}$$

where we have defined

$$Ch = \mathrm{diag}(\cosh \gamma_1\, \ell/2, \ \ldots, \ \cosh \gamma_K\, \ell/2) \tag{17}$$

$$Sh = [Ch^2 - 1]^{1/2} \tag{18}$$

$$Z = \mathrm{diag}\left(1, \ \bar{Y}_{02}^{-1}, \ \ldots, \ \bar{Y}_{0K}^{-1}\right) \quad . \tag{19}$$

In particular, we have

$$T = \mathrm{diag}(\cosh \gamma_i \ell) - jZ \quad Sh \quad X^{-1} \quad Ch \quad . \tag{20}$$

Imposing Floquet's theorem yields

$$L\begin{pmatrix} \underline{V} \\ \underline{I} \end{pmatrix} = e^{\theta\ell}\begin{pmatrix} \underline{V} \\ \underline{I} \end{pmatrix} \tag{21}$$

where $\underline{V}$ and $\underline{I}$ are k^{th} order column vectors representing the transverse electric and magnetic fields, respectively, at AA and CC (see Fig. 2). Making use of the reciprocity of the structure, (24) can be simplified into

$$T\underline{V} = \cosh \theta\ell \ \underline{V} \quad . \tag{22}$$

The matrix T is real, but not symmetric and therefore its eigenvalues are complex in general, in accordance with the well-known properties of periodic structures. As an example, the propagation characteristics of the first two eigenvalues of a corrugated waveguide with iris geometry

$c/a = 0.8$; $d/b = 0.6$; $\ell/a = 1/8$

was computed using $N = 7$, $K = 1,2,3,5,6$, and 7 in the band $1 \le \bar{\beta} \le 1.6$, which corresponds to the standard waveguide band. The results are displayed in Table 1.

The limiting cases of the capacitively and inductively loaded waveguides can be recovered from the general solution by setting $c/a = 1$ and $d/b = 1$.

Conclusions

From a Rayleigh-Ritz stationary formulation we have determined a lumped equivalent network model for the resonant iris in a rectangular waveguide.

The features of the method are as follows: (1) The elements of the equivalent circuit are frequency independent, (2) This network model has been used to determine the propagation properties of the periodic structure consisting of cascaded irises, (3) The propagation constant of the periodic structure is obtained as the solution of a matrix eigenvalue problem, and (4) The method is general, in that irises of arbitrary shape can be considered.

References

[1] T. E. Rozzi and W. F. G. Mecklenbräuker, "Wideband network modelling of interacting inductive irises and steps," IEEE Trans. Microwave Theory Tech., pp. 235-245, February 1975.

[2] T. E. Rozzi, "A new approach to the modelling of capacitive irises and steps in waveguide," to appear in the International Journal of Circuit Theory and Applications.

[3] R. E. Collin, Field Theory of Waveguides. New York: McGraw-Hill, 1961, ch. 8.

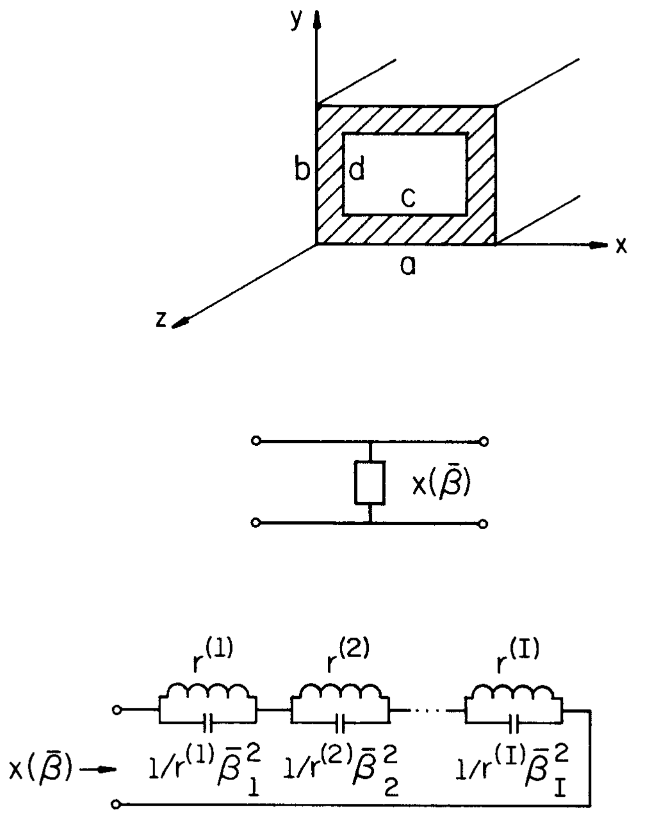

Fig. 1. Geometry of the iris and equivalent network.

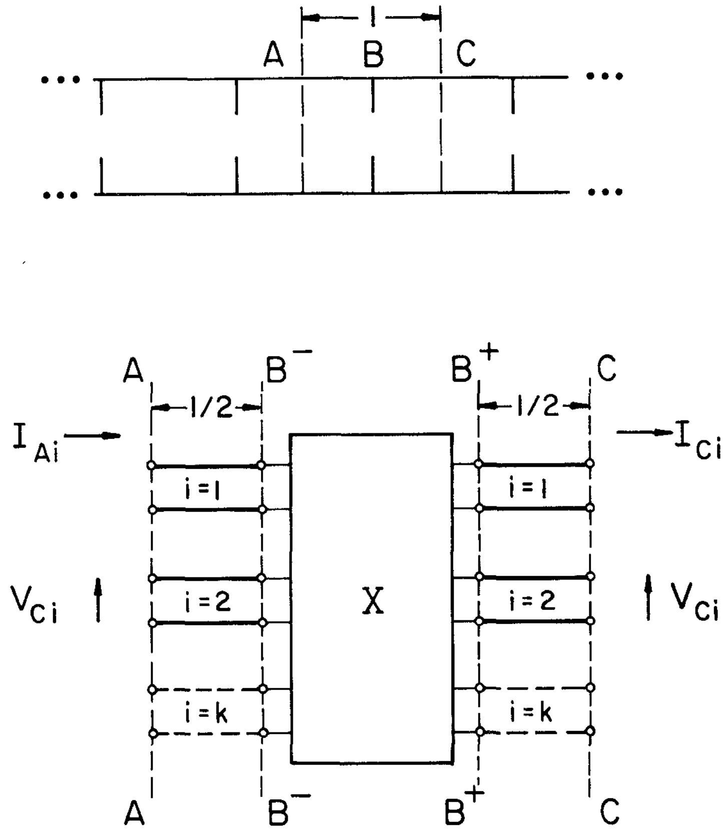

Fig. 2. Periodic structure plus unit cell plus equivalent network.

TABLE I

NORMALIZED PROPAGATION CONSTANT OF RECTANGULAR CORRUGATED WAVEGUIDE ($\bar{\theta} = \theta a$)

	$\bar{\beta}$							
	1.0	1.1	1.2	1.3	1.4	1.5	1.6	K
$\bar{\theta}_1$	j2.687	j3.220	j3.718	j4.192	j4.650	j5.098	j5.536	1
$\bar{\theta}_2$	——	——	——	——	——	——	——	
$\bar{\theta}_1$	j2.899	j3.393	j3.861	j4.313	j4.754	j5.185	j5.611	2
$\bar{\theta}_2$	13.251	13.132	13.002	12.860	12.705	12.536	12.354	
$\bar{\theta}_1$	j2.382	j2.934	j3.426	j3.887	j4.329	j4.757	j5.176	3
$\bar{\theta}_2$	13.323	13.191	13.056	12.911	12.754	12.586	12.403	
$\bar{\theta}_1$	j2.147	j2.694	j3.185	j3.645	j4.087	j4.516	j4.936	5
$\bar{\theta}_2$	12.830	12.767	12.716	12.661	12.597	12.523	12.437	
$\bar{\theta}_1$	j2.294	j2.825	j3.294	j3.747	j4.176	j4.599	j5.014	6
$\bar{\theta}_2$	15.358	14.995	14.835	14.660	14.486	14.307	14.120	
$\bar{\theta}_1$	j2.395	j2.903	j3.368	j3.808	j4.234	j4.651	j5.060	7
$\bar{\theta}_2$	15.188	14.732	14.525	14.356	14.191	14.020	13.840	